# Scalar Potential versus Vector Field

So far we've had to work quite hard to evaluate the electric field for various charge distributions. Whether we used discrete superposition (adding the fields of discrete charges), continuous superposition (integrating to sum the fields of all the tiny differential chunks of a charge density distribution) or Gauss's Law for the three highly symmetric charge distributions, there is some possibly fairly serious calculus and geometry involved.

A lot of the difficulty comes from the *vector* nature of the electric field. We cannot just add one number for each source point, we have to add 2-3 numbers (each field component) separately, and those components often contain sines and cosines of angles to represent the geometry of the arrangement. So far we've gotten by, but it hasn't always been easy.

Fortunately, we are now ready to pitch this vector description in favor of a scalar description. The scalar description will in some sense be ``more fundamental'' than the vector description, in that one will be able to obtain the vector description from the scalar description by (relatively simple) differentiation. The description is also easily connected to things we are very interested in later - power, electric current, energy.

To get you there we start by remembering our definition of the *change in potential energy* associated with moving a particle from position $\vec{x}_0$ to $\vec{x}_1$ against a given *conservative force*:

$$\Delta U = -\int_{\vec{x}_0}^{\vec{x}_1} \vec{F} \cdot d\vec{l} \qquad (1)$$

The particular force we are interested in is, of course, the electrostatic force produced by the electrostatic field:

$$\vec{F} = q\vec{E} \qquad (2)$$

or

$$\Delta U = -q\int_{\vec{x}_0}^{\vec{x}_1} \vec{E} \cdot d\vec{l}. \qquad (3)$$

If we define the change in the *electric potential* in terms of the change in the electric potential energy of a given charge, per unit charge (exactly as we did for the field) we get:

$$\Delta V = \lim_{q\to 0} \frac{\Delta U}{q} = -\int_{\vec{x}_0}^{\vec{x}_1} \vec{E} \cdot d\vec{l} \qquad (4)$$

which is simply lovely. This is now a *scalar* field (function on all the coordinates of space) from which we can easily find the (change in the) potential energy of a given charge $q$:

$$\Delta U = q\Delta V \qquad (5)$$

or (by inverting the integral definition) the electric field:

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}\right) = -\vec{\nabla}V. \quad (6)$$

This definition is perfectly adequate, but it suffers from two minor problems. The most important one is that it requires us to *first* find the field and *then* find the potential, which is exactly the opposite of what we want to do. The other is that we've somehow misplaced Gauss's Law and we'd like to see what it looks like in terms of the potential instead of the field. We'll handle these problems in a moment. First, though, let us take care of a bit of important trivia (gotta love those oxymorons).

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